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A Function-based Fuzzy PID (F-FPID) Controller

Yehya Dadam, Eldaw Eldukhri

Abstract— This paper describes the concept of a function-based fuzzy proportional-integral-derivative (F-FPID) controller. An important feature of the proposed controller is that its structure comprises only four fuzzy rules each with two antecedents and two consequents. It performs proportional-derivative (PD) and integral (I) control actions using fuzzy elements rather than analytically using delay loops as is the case in input-based FPID (I-FPID). To assess its viability, the developed controller was used to control a bench-scale pH process. The experimental results demonstrated satisfactory performance of the proposed controller in maintaining the pH value at prescribed levels.

Index Terms— Fuzzy logic control, function-based fuzzy PID, pH control process.

1 Introduction

ALTHOUGH many sophisticated control theories and techniques have been developed in the last few decades (Middelton, Goodwin, Hill and Mayne 1988; Branicky, Borkar and Mitter 1998; Morari and Lee 1999; Syafiie, Tadeo, Martinez and Alvarez 2011; Astrom and McAvoy1992; Loh, Looi and Fong 1995), PID controllers continue to be the most commonly used in industrial processes (Cominos and Munro 2002). This is because they have simple structure and are relatively easy to implement by great majority of practitioners and automatic control designers. However, they are usually not effective if the processes have inherently intractable characteristics such as high order and nonlinearities (Visioli 2001), as is the case with pH control process. These difficulties meant that various types of modified conventional PID controllers such as auto-tuning and adaptive PID controllers were developed lately (Astrom, Hagglund, Hang and Ho 1993; Ho, Hang and Cao 1995).

Nonetheless, benefiting from the recent developments in evolutionary computing, many researchers have focused their research on improving the performance of PID controllers and extending their applicability to more complex systems (Mann, Hu and Raymond 2001; Mohan and Sinha 2008). In particular, fuzzy logic control (FLC) has found many successful industrial applications and demonstrated significant performance improvements (Baogang, George, and Raymond 1999; Zhu, Toncich, Nagarajah and Romanski 1997; Khan and Rapal 2006; Zhao, Tomizuka and Isaka 1993). Nevertheless, fuzzy controller design remains an imprecise process because there are insufficient analytical design techniques available in contrast with the well developed linear control theories. Despite this, in fuzzy systems, the way that the control knowledge is built gives flexibility to the controlled process to act in a linear or non-linear manner depending on the operating conditions.

A large amount of research has been carried out on designing fuzzy PID controllers (FPID) (Baogang, George, and Raymond 1999; Zhu, Toncich, Nagarajah and Romanski 1997). These fuzzy controllers can be classified into three types, namely, direct action (DA); gain scheduling (GS); and combination of DA and GS types. In GS type controllers, fuzzy inference is used to calculate the individual PID gains and the inference is either error driven self-tuning (Zhao, Tomizuka and Isaka 1993) or performance-based supervisory tuning (Silva 1995). The majority of FPID applications belong to the DA type where the FPID is placed within the feedback control loop and computes the PID actions through fuzzy inference (Khan and Rapal 2006; George, Baogang and Raymond 1999). Direct action FPID (DAFPID) structures have been reported with one, two or three inputs (error, change of error and rate of change of error) (Mann, Hu and Raymond 1997; Mann, Hu and Gosine 1999). In DAFPID, the integration and derivation are executed entirely outside of the fuzzy controllers. In other words, DAFPID does not use any fuzzy system to carry out either the fuzzy integral or the fuzzy derivative actions. So the fuzzy system only performs the non-linear amplification associated with the PID's three control actions. Consequently, DAFPID controllers are input-based FPID controllers (I-FPID) rather than function-based FPID controllers (F-FPID).

This paper describes a function-based FPID controller that is designed using two fuzzy control elements, PD and I to control a bench-scale pH process. The controller has the following advantages over I-FPID controllers:

1. It has a simple rule based structure of four rules each with two antecedents and two consequents.
2. It performs proportional derivative (PD) and integral control actions using fuzzy elements rather than analytically using delay loops as is the case in I-FPID (Zhu, Toncich, Nagarajah and Romanski 1997; Silva 1995; Mann, Hu and Gosine 1999).
3. The two output control actions (PD and I) can be tuned independently unlike I-FPID controllers (Mann, Hu and Raymond 1997; Mann, Hu and Gosine 1999).

The remainder of the paper is organized as follows. Section 2 describes the F-FPID structure. In section 3, the design procedure for the F-FPID controller is outlined. Section 4 presents the experimental results. Conclusions are given in section 5.

2 F-FPID Structural Elements

Linear PID controllers can be classified into different categories with respect to the positioning of the three terms in the

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closed loop control system. The cascade form PID controller is commonly employed and its output in Laplace-form is given by:

$$U_{PID}(s) = \left(K_p + sK_d + \frac{K_i}{s} \right) E(s) \quad (1)$$

where K_p, K_d and K_i are the proportional, derivative and integral gains respectively, and E(s) is the error signal in Laplace-form. The PID controller's output can be rewritten in a discrete form as follows:

$$U_{PID}(nT) = U_P(nT) + U_D(nT) + U_I(nT) \quad (2)$$

where T > 0 is the sampling time and n=1, 2, 3... By using the backward difference rule for differentiation and the trapezoid integration rule for integration, each term of (2) can be defined as follows:

$$U_P(nT) = K_P e(nT) \quad (3)$$

$$U_D(nT) = K_D \Delta e(nT) \quad (4)$$

$$U_I(nT) = U_I((n\text{-}1)T) + K_I \Delta U_I(nT) \quad (5)$$

where

$$e(nT) = Y_d(nT) - Y(nT) \quad (6)$$

$$\Delta e(nT) = e(nT) - e((n\text{-}1)T) \quad (7)$$

$$\Delta U_I(nT) = e(nT) + e((n\text{-}1)T) \quad (8)$$

where *e(nT)*, *Yd(nT)* and *Y(nT)* are the signal error, the feedback response signal and the desired response signal all at $t = nT$ and $K_P = K_p$, $K_D = K_d/T$ and $K_I = K_iT/2$. Substituting (3), (4) and (5) into (2) gives:

$$U_{PID}(nT) = K_P e(nT) + K_D \Delta e(nT) + U_I((n\text{-}1)T) + K_I \Delta U_I(nT) \quad (9)$$

As it can be seen from (9), the output of the PID controller in absolute form is achieved by employing the two input error variables e(nT) and e((n-1)T). This can be rewritten as follows:

$$U_{PID}(nT) = f_{PD}(e(nT), e(n\text{-}1)T) + f_I(U_I((n-1)T), e(nT), e((n\text{-}1)T)) \quad (10)$$

where f_{PD}, f_I are the proportional-derivative and integration functions to be implemented employing fuzzy inference and $U_I((n-1)T)$ is the previous integration action. The functions f_{PD}, f_I can be designed and implemented using two-input function based fuzzy elements. Then, the final outputs of both functions are summed to form the overall F-FPID controller as depicted in fig. 1. The design and implementation of the two functions f_{PD}, f_I is explained in the following section.

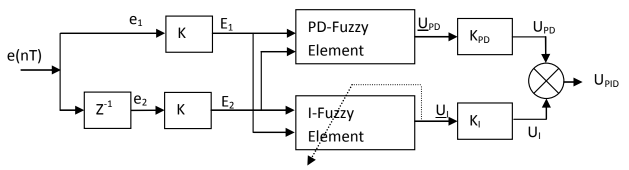

Fig. 1. F-FPID controller

3 Design Procedure for a Function Based Fuzzy Controller

In (10) it can be seen that the proportional-derivative and integral functions f_{PD}, f_I are respectively functions of the two error variables *e(nT)* and *e((n-1)T)*. After normalising within the range [-1, 1], the two error variables E_1 and E_2 (see Fig. 1) can be expressed as follows:

$$E_1 = Ke(nT) = Ke_1 \quad (11)$$

$$E_2 = Ke((n\text{-}1)T) = Ke_2 \quad (12)$$

where *K* is the input scaling factor. Because the two input variables are of the same nature, their input universes will be designed similarly. The universes of discourse of each input variable are identically partitioned by employing N symmetrical triangular membership functions with a 50% overlap as shown in Fig. 2. The membership functions on the right and left ends of the range are right-angled triangles. In turn, the input variable range is divided into M = N -1 ranges each of length 2L.

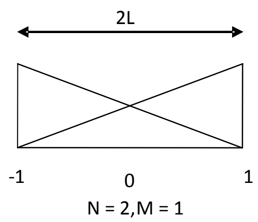

Fig. 2. Universe of discourse for F-FPID input variables E1, E2

The universe of discourse of the output is divided into 2M-1 output zones. The total number of membership functions defined on the output universe of discourse is equal to 4(2M-1). By employing a two-input control element, a fuzzy rule base with $(N\text{-}1)^2$ input operating zones can be built depending on the two dimensional input space. One output zone is employed for each input zone. Four fuzzy rules are generated for each input operating zone. No more than one input zone is allowed to fire at one time. Therefore, $4(N\text{-}1)^2$ fuzzy rules with two antecedents and two consequents are generated to form F-FPID controller. The input operating zone is specified according to the position of the crisp inputs in their corresponding input universes. As well as this, within the specified input zone, one of eight input conditions (IC_1-IC_8), as shown in Fig. 3, can be specified [21]. In the following section, an analytical solution for the function based control elements f_{PD} will be achieved.

3.1 Function based Proportional-Derivative Fuzzy control element (F-FPD)

Two membership functions (N=2 and M=1) are considered for each input universe for the normalised input variables E1 and E2 as presented in Fig. 4. These two membership function universes represent one of the input zones mentioned in the previous section.

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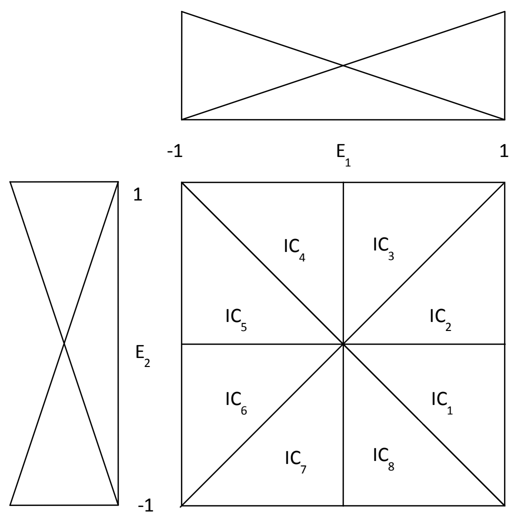

Fig. 3. Possible input conditions for each input zone

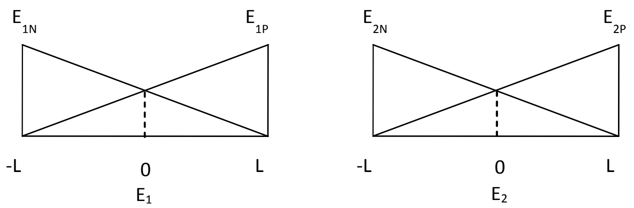

Fig. 4. Input universes used to derive the F-FPID output analytically

Since the number of output zones is 2M-1, one output zone resulted with four membership functions. The output universe for the normalised output U_{PD} is shown in Fig. 5. These four membership functions represent one output zone; the distances between the centres of the membership functions are shown in the figure.

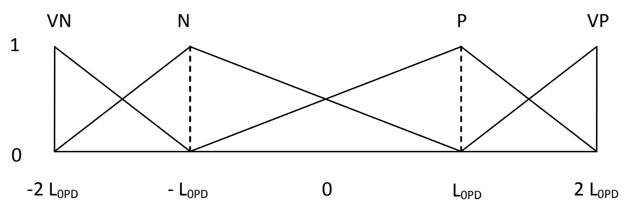

Fig. 5. Universe of discourse for F-FPID input variables E1, E2

As mentioned previously, only one input operating zone is considered and four fuzzy rules are generated as follows:

Rule1: IF E_1 is E_{1P} AND E_2 is E_{2P} THEN $\underline{U}_{PD}$ is P
Rule2: IF E_1 is E_{1P} AND E_2 is E_{2N} THEN $\underline{U}_{PD}$ is VP
Rule3: IF E_1 is E_{1N} AND E_2 is E_{2P} THEN $\underline{U}_{PD}$ is VN
Rule4: IF E_1 is E_{1N} AND E_2 is E_N THEN $\underline{U}_{PD}$ is N

where [E_{1P}, E_{1N}], [E_{2P}, E_{2N}] and [P, VP, N, VN] are fuzzy terms of the normalised input variables E_1 and E_2 and the normalised output variables $\underline{U}_{PD}\ (nT)$ respectively. These rules are generated conventionally following the approach adopted in [22]. Mamdani's Min-max method is employed to infer the fuzzy output of the control element. As well as this, the centre average defuzzification method [23] is used to calculate the control element crisp output. The fuzzy output will be a trapezoid whose height (h) is equal to the membership degree resulting from the min-operator through the fuzzy inference. Based on the inference employed and the height value (h) for each of the four rules under the eight input conditions, the fuzzy output is listed in Table 1. The analytical solution of the fuzzy control element proportional-derivative function $f_{PD}(e_1, e_2)$ is found to be as follows for the input conditions IC_1, IC_2, IC_5, and IC_6 [3]:

$$f_{PD}(e_1,e_2) = \frac{KL_{O_{PD}}}{2L\text{-}Ke_1} e_1 + \frac{KL_{O_{PD}}}{4L\text{-}2Ke_1}(e_1 - e_2) \quad (13)$$

(13) can be rewritten as follows:

$$\underline{U}_{PD}(nT) = K_{NP_1}\, L_{O_{PD}} e_1 + K_{ND_1}\, L_{O_{PD}}(e_1 - e_2) \quad (14)$$

where, using (11)

$$K_{NP_1} = \frac{K}{2L - Ke_1} = \frac{K}{2L\text{-}E_1} = 2K_{ND_1} \quad (15)$$

Similarly, the output of the function based proportional derivative control element for the input conditions IC3, IC4, IC7, and IC8 is given by:

$$\underline{U}_{PD} = \frac{KL_{O_{PD}}}{2L\text{-}Ke_2} e_1 + \frac{KL_{O_{PD}}}{4L\text{-}2Ke_2}(e_1 - e_2) \quad (16)$$

Consequently, the output of the function based proportional derivative control element for those input conditions is given by:

$$\underline{U}_{PD}(nT) = K_{NP_2}\, L_{O_{PD}} e_1 + K_{ND_2}\, L_{O_{PD}}(e_1 - e_2) \quad (17)$$

where, using (12)

$$K_{NP_2} = \frac{K}{2L - Ke_2} = \frac{K}{2L\text{-}E_2} = 2K_{ND_2} \quad (18)$$

where $K_{NP_1}, K_{ND_1}, K_{NP_2}$ *and* K_{ND_2} are non-linear proportional and derivative gains calculated in terms of e_1 and e_2 at each sampling interval according to the input conditions . The finer the partition is the more input and output operating zones are required. The same set of rules can be used for different input and output zones, but the fuzzy terms in both the antecedents and consequents must match the input and output operating zones. In the next section the function based integration fuzzy control element will be explained as introduced in [24].

3.2 Function based Integral Fuzzy control element (F-FI)

The conventional integral action as shown in (5) consists of two parts. The first part represents the accumulated control action starting from initial condition $U_I((n\text{-}1)T)$; the second part is the incremental output of the controller $\Delta U_I(nT)$. Consequently, the output of the function based integration element must have the same structure. It has been reported that to implement the integration initial condition, the centres of the output universe membership functions are shifted after the

TABLE 1
PROPORTIONAL-DERIVATIVE FUZZY ELEMENT

	Rule1			Rule2			Rule3			Rule4			Output of the fuzzy element
Input conditions	Input MF With Min h	Output MF centre	H value	Input MF With Min h	Output MF centre	H value	Input MF With Min	Output MF centre	H value	Input MF With h	Output MF centre	H value	
IC_1 - IC_2	E_{2P}	Lo	$\frac{Ke_2+L}{2L}$	E_{2N}	2Lo	$\frac{L-Ke_2}{2L}$	E_{1N}	-2Lo	$\frac{L-Ke_1}{2L}$	E_{1N}	-Lo	$\frac{L-Ke_1}{2L}$	$\frac{KL_o}{2L-Ke_1}e_1+\frac{KL_o}{4L-2Ke_1}(e_1-e_2)$
IC_3 - C_4	E_{1P}	Lo	$\frac{Ke_1+L}{2L}$	E_{2N}	2Lo	$\frac{L-Ke_2}{2L}$	E_{1N}	-2Lo	$\frac{L-Ke_1}{2L}$	E_{2N}	-Lo	$\frac{L-Ke_2}{2L}$	$\frac{KL_o}{2L-Ke_2}e_1+\frac{KL_o}{4L-2Ke_2}(e_1-e_2)$
IC_5 - IC_6	E_{1P}	Lo	$\frac{Ke_1+L}{2L}$	E_{1P}	2Lo	$\frac{Ke_1+L}{2L}$	E_{2P}	-2Lo	$\frac{Ke_2+L}{2L}$	E_{2N}	-Lo	$\frac{L-Ke_2}{2L}$	$\frac{KL_o}{2L-Ke_1}e_1+\frac{KL_o}{4L-2Ke_1}(e_1-e_2)$
IC_7 - IC_8	E_{2P}	Lo	$\frac{Ke_2+L}{2L}$	E_{1P}	2Lo	$\frac{Ke_1+L}{2L}$	E_{2P}	-2Lo	$\frac{Ke_2+L}{2L}$	E_{1N}	-Lo	$\frac{L-Ke_1}{2L}$	$\frac{KL_o}{2L-Ke_2}e_1+\frac{KL_o}{4L-2Ke_2}(e_1-e_2)$

k^{th} time step by a distance $C=\sum_{n=0}^{k-1}\Delta U_I(nT)$ [24]. The output universe of discourse is partitioned as shown in Fig. 6.

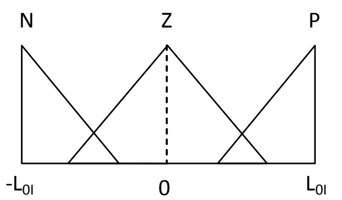

Fig. 6. Output universe used to derive F-FI output analytically

The shifting process, which represents the memory of the fuzzy integration element, is shown in Fig. 7.

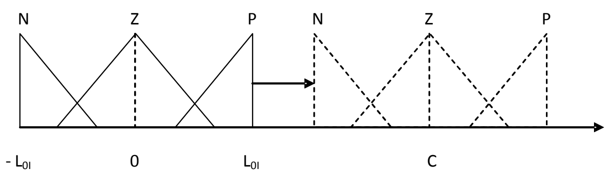

Fig. 7. Shifting process of the integration fuzzy element. $C=\sum_{n=0}^{k-1}\Delta U_I(nT)$

The same procedure as in the previous section is pursued for partitioning the input universes of the two error variables. Four fuzzy rules are generated as follows:

Rule1: IF E_1 is E_{1P} AND E_2 is E_{2P} THEN $\Delta \underline{U}_I$ (nT) is P
Rule2: IF E_1 is E_{1P} AND E_2 is E_{2N} THEN $\Delta \underline{U}_I$ (nT) is Z
Rule3: IF E_1 is E_{1N} AND E_2 is E_{2P} THEN $\Delta \underline{U}_I$ (nT) is Z
Rule4: IF E_1 is E_{1N} AND E_2 is E_{2N} THEN $\Delta \underline{U}_I$ (nT) is N

where [E_{1P}, E_{1N}], [E_{2P}, E_{2N}] and [P, Z, N] are fuzzy terms of the normalised input variables E1and E2 and the normalised output variables $\Delta \underline{U}_I(nT)$ respectively. More details on how the fuzzy rules were generated can be found in [18]. In a similar way, by employing the same inference and the same defuzzification method used in the previous section, the output of the integration fuzzy element can be obtained. Based on the inference employed and the value (h) of each fuzzy rule under the eight input conditions, the fuzzy output and the value (h) of each fuzzy rule under the four input conditions are listed in Table 2. The analytical solution to the fuzzy control element integration function $f_I(\underline{U}_I((n\text{-}1)T), e_1, e_2)$ in (10) is found to be as follows for the input conditions IC1, IC2, IC5, and IC6:

$$f_I\left(\underline{U}_I((n-1)T), e_1, e_2\right) = C + \left(KL_{O_I}/4L - 2Ke_1\right)\left(e_1 + e_2\right) \quad (19)$$

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Similarly, for the input conditions IC_3, IC_4, IC_7, and IC_8, $\underline{U}_I(nT)$ will be:

$$\underline{U}_I(nT) = C + \frac{KL_{O_I}}{4L - 2Ke_2}(e_1 + e_2) \quad (20)$$

In general, the output of the function based integration control element can be rewritten using (19) and (20) as follows:

$$\underline{U}_I(nT) = C + K_{NI} L_{O_I}(e_1 + e_2) \quad (21)$$

where $K_{NI} = K/(4L - 2Ke_1) = K/(4L - 2E_1)$ is the non-linear integral gain for the input conditions IC_1, IC_2, IC_5, and IC_6 and $K_{NI} = K/(4L - 2Ke_2) = K/(4L - 2E_2)$ for the input conditions IC_3, IC_4, IC_7, and IC_8.

TABLE 2

INTEGRAL FUZZY ELEMENT

	Rule1			Rule2			Rule3			Rule4			Output of the fuzzy element
Input conditions	Input MF With Min	Output MF centre	H value	Input MF With Min	Output MF centre	H value	Input MF With Min	Output MF centre	H value	Input MF With Min h	Output MF centre	H value	
IC_1 - IC_2	E_{2P}	$C+L_o$	$\frac{Ke_2+L}{2L}$	E_{2N}	C	$\frac{L-Ke_2}{2L}$	E_{1N}	C	$\frac{L-Ke_1}{2L}$	E_{1N}	$C-L_o$	$\frac{L-Ke_1}{2L}$	$C+\frac{KL_o}{4L-2Ke_1}(e_1+e_2)$
IC_3 - IC_4	E_{1P}	$C+L_o$	$\frac{Ke_1+L}{2L}$	E_{2N}	C	$\frac{L-Ke_2}{2L}$	E_{1N}	C	$\frac{L-Ke_1}{2L}$	E_{2N}	$C-L_o$	$\frac{L-Ke_2}{2L}$	$C+\frac{KL_o}{4L-2Ke_2}(e_1+e_2)$
IC_5 - IC_6	E_{1P}	$C+L_o$	$\frac{Ke_1+L}{2L}$	E_{1P}	C	$\frac{Ke_1+L}{2L}$	E_{2P}	C	$\frac{Ke_2+L}{2L}$	E_{2N}	$C-L_o$	$\frac{L-Ke_2}{2L}$	$C+\frac{KL_o}{4L-2Ke_1}(e_1+e_2)$
IC_7 - IC_8	E_{2P}	$C+L_o$	$\frac{Ke_2+L}{2L}$	E_{1P}	C	$\frac{Ke_1+L}{2L}$	E_{2P}	C	$\frac{Ke_2+L}{2L}$	E_{1N}	$C-L_o$	$\frac{L-Ke_1}{2L}$	$C+\frac{KL_o}{4L-2Ke_2}(e_1+e_2)$

Based on the above description, it can be seen that the fuzzy system can approximate an integration control function with non-linear gain K_{NI}. The finer the partition is, the larger the number of input and output zones that is required.

The same set of rules can be used for different input and output zones, but the fuzzy terms in both the antecedents and consequents must match the input and output operating zones.

3.3 F-FPID total output

The final output of the F-FPID (see Fig. 1) is calculated as a summation of both outputs of the two fuzzy elements (PD, I) described respectively in (17) and (21) as follows:

$$U_{PID} = K_{PD}\underline{U}_{PD} + K_I \underline{U}_I \quad (22)$$

where K_{PD} and K_I are the output scaling factors. Subsequently, the final output of the F-FPID can be tuned through linear and non-linear gains. Comparing the output of the F-FPID with the output of a one-input I-FPID [22], it can be seen that the output of the I-FPID can be tuned in a similar way to the F-FPID through one non-linear gain and three independent linear gains. Therefore, the differentiation and the integration functions are achieved based on the output of the proportional element not on the error as in F-FPID. Hence, the I-FPID performs only as a fuzzy proportional device and then the differentiation and integration are done analytically based on the fuzzy proportional element output. From an adaptive tuning point of view the modification depends on the proportional action, while in the F-FPID, free parameters can be tuned based on the output of both PD and I.

4 F-FPID Application and Results

To show the efficiency of the proposed F-FPID controller, a highly non-linear and time varying system was controlled using such a controller. This system (depicted in Fig. 8) is an online pH control process which is found in a variety of industries including wastewater treatment, pharmaceuticals, bio-

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technology and chemical processing [6]. In these applications the pH value must be maintained within stringent limits. However, high performance and robust pH control is often difficult to achieve due to the nonlinear and time-varying characteristics of the process [25]. The main aim is to control the pH value of the reactor solution. However, since the level of the reactor plays an important role in changing the features and the initial conditions of the process, both the pH value and the solution level were controlled simultaneously. The input variables of the pH controller are the current and previous errors between the pH set points and the actual pH value in the reactor, measured by a pH sensor, while the output variable is the voltage applied to either the Acid or the Alkali pumps. The input variables of the level controller are the current and previous errors between the level set points and the actual levels in the reactor, measured by the level sensor, while the output variable is the voltage applied to the recycled solvent pump.

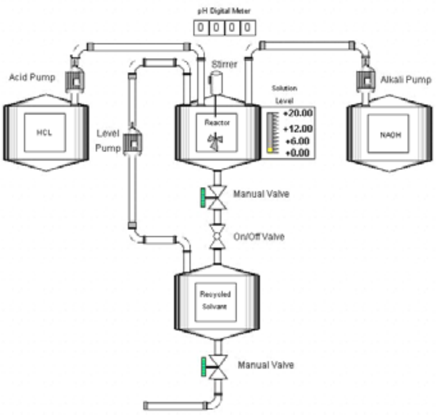

Fig. 8 pH control process

Following the F-FPID design procedure explained earlier, the input spaces of the two fuzzy control elements PD and I were partitioned into two membership functions (M=2). This resulted in one input operating zone. The output universes of the two fuzzy control elements were partitioned into four membership functions. The distances $L_{O_{PD}}$ and L_{O_I} were set at 0.5 and 0.08, respectively. As a result, four fuzzy rules, each with two antecedents (E_1, E_2) and two consequents $(\underline{U}_{PD}, \Delta\underline{U}_I(nT))$ were generated for the single input operating zone. The F-FPID rule base for the single input operating zone is as follows:

Rule 1 : IF E_1 is E_{1P} AND E_2 is E_{2P} THEN $\underline{U}_{PD}$ is P AND $\Delta\underline{U}_I$ (nT) is P
Rule 2 : IF E_1 is E_{1P} AND E_2 is E_{2N} THEN $\underline{U}_{PD}$ is VP AND $\Delta\underline{U}_I$ (nT) is Z
Rule 3 : IF E_1 is E_{1N} AND E_2 is E_{2P} THEN $\underline{U}_{PD}$ is VN AND $\Delta\underline{U}_I$ (nT) is Z
Rule 4 : IF E_1 is E_{1N} AND E_2 is E_{2N} THEN $\underline{U}_{PD}$ is N AND $\Delta\underline{U}_I$ (nT) is N

The parameters K_{PD} and K_I were tuned in a heuristic way to give as good a performance as possible. The parameter K_{PD} was assigned the same value (K_{PD}=80) for the different operating ranges of pH. However, due to the high non-linearity of the pH process, different integration gains (K_I=0.1, 0.25 and 0.5) were used to accommodate the steady state errors associated with the dissimilar operating ranges of pH. The output of the controller was used to determine which chemical, acid (HCL) or alkali (NaOH), should be pumped into the reactor. Although the level of the reactor could have been controlled using a simpler controller such as a conventional P or PI, it was also controlled using the F-FPID controller but with different parameters. Because the focus of this work was on controlling the pH values, the solution levels were not presented. The control scheme was constructed as shown in Fig. 9.

To demonstrate the effectiveness of the proposed F-FPID, different set points for the pH value were used as shown in Fig. 10, 11 and 12.

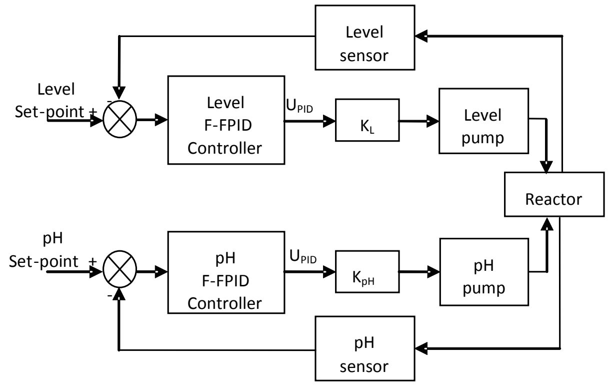

Fig. 9. Control system loop employed for pH process

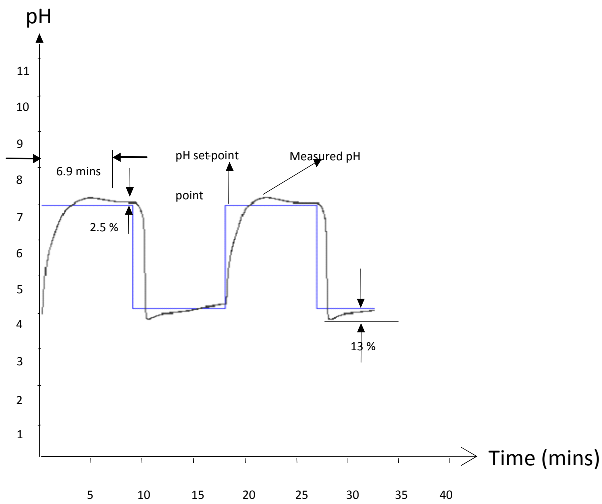

Fig. 10. pH control using F-FPID with parameters: K_{PD}=80; K_I=0.1

5 CONCLUSIONS

In this paper the design procedure for a new function based fuzzy PID controller (F-FPID) was outlined. The proposed F-FPID controller consists of two fuzzy control elements PD and I. A rule base of four fuzzy rules, each with two antecedents and two consequents, was generated. Based on that rule base, analytical solutions of the two fuzzy elements (PD and I) were derived proving that those two fuzzy elements and the generated rule base can approximate proportional-derivative and integral functions. It was proven that the final output of F-FPID represents a PID-like controller with a non-linear control policy. The advantage of F-FPID over I-FPID controller is that it performs differentiation and integration using purely fuzzy elements rather that utilising delay loops as in I-FPID controllers. Unlike I-FPID, the two control actions of F-FPID can be tuned independently to achieve the required performance. Finally, to evaluate the new concept of F-FPID, a highly nonlinear pH system was controlled over different ranges. The proposed controller proved itself capable of handling such a system.

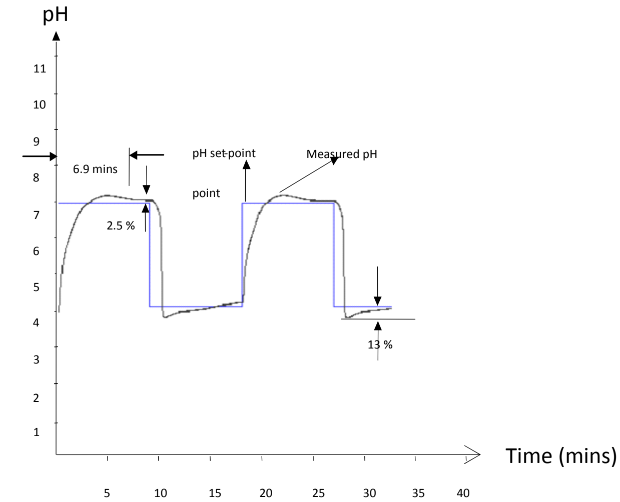

Fig. 11. pH control using F-FPID with parameters: K_{PD}=80; K_I=0.25

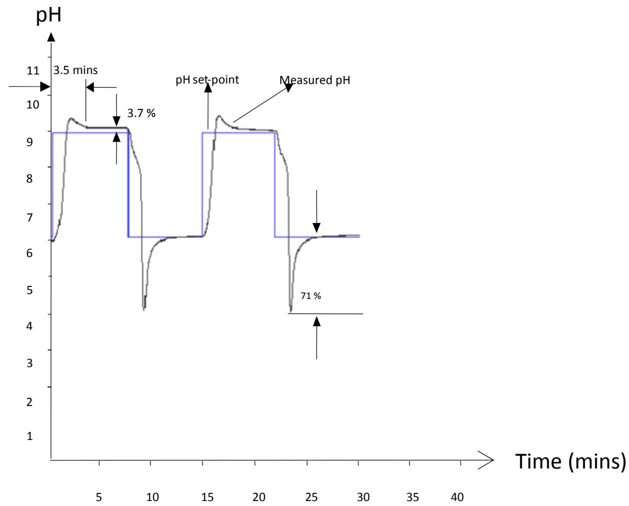

Fig. 12. pH control using F-FPID with parameters: K_{PD}=80; K_I=0.5

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